Dark Matter NASA Conference

Suggested Grade Level(s) 9 - 12
Estimated class time 2 periods plus extensions. In addition, allow one class period for reading the Cosmic Times if they have not yet read the article relating to dark matter.

Summary

Students will explore dark matter by first calculating the escape velocity of planets in our solar system and then applying the same method to calculate the escape velocity for NGC 2300 group. They will compare the group’s escape velocity to the velocity of the gas in the group as determined by analysis of the X-ray spectra. Finally, they will suggest reasons why the escape velocities are higher than possible with the amount of matter that is visible.

Objectives

• Students will apply the Law of Conservation of Energy to calculate escape velocity.
• Students will explain the findings of ROSAT and hypothesize about dark matter.

National Science Standards

• NS.9-12.1 SCIENCE AS INQUIRY
  As a result of activities in grades 9-12, all students should develop
  o Abilities necessary to do scientific inquiry
  o Understandings about scientific inquiry

• NS.9-12.2 PHYSICAL SCIENCE
  As a result of their activities in grades 9-12, all students should develop an understanding of
  o Motions and forces
  o Conservation of energy and increase in disorder
  o Interactions of energy and matter

• NS.9-12.4 EARTH AND SPACE SCIENCE
  As a result of their activities in grades 9-12, all students should develop an understanding of
  o Origin and evolution of the universe

Knowledge Prerequisite

• Students should be familiar with the article on Dark Matter from the 1965 Edition of Cosmic Times.
• Students should be familiar with the Law of Universal Gravitation.
• Students should be able to work with the formulas for kinetic and potential energy and the Law of Conservation of Energy.
Teacher Background/Notes

This is the link to the original news release that provided the background for the 1993 Cosmic Times article “Dark Matter Hunt Heats Up”:
http://www.nasa.gov/home/hqnews/1993/93-001.txt

Article with detailed look at analysis of X-ray data:
http://www.astronomy.pomona.edu/astro121/mulchahey.araa.pdf

The teacher should be familiar with the concept of escape speed for a planet or galaxy. This is derived from the Law of Conservation of Energy and Newton’s Law of Universal Gravitation.

If you toss a ball into the air it will slow down until it momentarily stops and then it will fall back into your hand, gaining speed on the way down. The “up” half of the trajectory is symmetrical to the “down” half.

If you fire the rockets for the space shuttle, it will slow down until it reaches some orbital speed. On the return trip to earth, the space shuttle speeds up as it returns to earth. Sir Isaac Newton understood that the force which keeps our natural satellite, the moon, in orbit is the same as the force that causes an apple to fall from a tree to the ground.

Although gravitational force is the weakest of the fundamental forces of nature, it extends over the largest distances. Here on earth we are under the influence of the earth’s gravity; however, we are also under the influence of the gravity of the sun as we orbit the sun, and the gravity of our galaxy and even the local group of galaxies. If we were to travel in a space ship on a journey throughout our solar system, it would be necessary to leave earth with a speed of 11.2 km/s in order to escape from our earth’s gravity and in order not to be locked into an orbit around the earth. This speed is calculated from Newton’s Law of Universal Gravitation and the Law of Conservation of Energy.

Explanation of derivation of escape velocity: See Appendix A

Explanation of how X-ray data shows the temperature of the gas:

Some resources – see Appendix B
http://hyperphysics.phy-astr.gsu.edu/Hbase/wien.html
http://hyperphysics.phy-astr.gsu.edu/Hbase/wien.html#c2

Explanation of calculating the energy radiated from the temperature:

Some resources – see Appendix B
http://hyperphysics.phy-astr.gsu.edu/Hbase/thermo/stefan.html#c1

Background information about the ROSAT Mission
http://heasarc.nasa.gov/docs/rosat/rosgof.html
http://heasarc.nasa.gov/docs/rosat/rosat.html
The ROSAT Mission
(1990-1999)

ROSAT, the Röntgen Satellite, was an X-ray observatory developed by institutes in Germany, the United States, and the United Kingdom. The satellite was proposed by the Max-Planck-Institut für extraterrestrische Physik (MPE). It was designed, built and operated in Germany. The satellite was launched by the United States on June 1, 1990 and the mission ended on February 12, 1999.

ROSAT was able to analyze X-rays from Jupiter and the Comet Hyakutake in our solar system, and from many different stars. ROSAT observed star clusters, protostars, and binary star systems. ROSAT also observed the remnants of supernova explosions, which provide information on the origin of the elements and the evolution of stars.

ROSAT’s X-ray picture of the NGC 2300 group of galaxies revealed a large cloud of hot gas in the area of three galaxies that was emitting in the X-ray range of frequencies. The gases were heated to approximately $10^7$ K by their fall toward the center of the galactic group. The cloud extended to a diameter of 1.3 million light-years and has a mass equal to 500 billion times that of the Sun.

"A cloud like this would have dissipated into space long ago, leaving nothing for us to detect, unless it was held together by the gravity of an immense mass," Dr. Mushotzky said. "The mass required to restrain the cloud is 12 to 25 times greater than the mass of the three galaxies that are present."


Materials

- A small ball that can easily be tossed into the air and caught again.
- Scientific calculators – graphing calculators are fine, but not required.
- Computers

Procedure:

I. Engagement

Give your students the following scenario:

You are a student who has been chosen as your state’s representative to a NASA Conference for Young Scientists to be held in Washington, D.C. The time is summer 1993.

At this conference you are introduced to ideas such as:

- X-ray observations determine the temperature of the gas.
- Temperature is related to average kinetic energy of the gas molecules.
- From kinetic energy, we can get an average velocity of the gas.
- Compare that velocity to escape velocity computed from the visible mass:
• \( V_{\text{gas}} > V_{\text{escape}} \)
• In order for the gas to be present, there must be additional unseen mass to hold it in.
• **There is evidence for dark matter!**

You are expected to return to your town and write an article for the school newspaper in September.

What information will you need from your physics teacher at home?
What questions must you ask the NASA scientists?
What research must you do on your own to write an article for your classmates?

At the end of this lesson you will be able to write this article.

You will do some calculations with the help of your physics teacher.
You will use the internet to search for information from NASA and other sources.
You will write the article for the newspaper that explains NASA findings from the period and details your experiences at the conference in Washington, DC.

**II. Exploration**

Throw a ball into the air and ask students to describe its motion.
Possible responses include:
  - Slows down as it rises, momentarily stops, falls and speeds up coming down
  - Same speed leaving your hand as it has when it reaches your hand again
  - Same time to rise to top as takes to fall.
  - Acceleration due to gravity is 9.80 m/s\(^2\)

Ask:
How hard and how fast would I have to throw this ball so that it would rise and slow down but NEVER stop and return to earth?

\((11.2 \text{ km/s})\)

This speed is called the escape speed.
Today we will explore the escape speed of the earth and a group of galaxies. We will try to explain why the hydrogen gas in that galaxy does not “escape.”
If we assume that the particle has sufficient speed and kinetic energy to escape from gravity, we can set the kinetic energy of the particle equal to the potential energy and derive a value that gives us:

\[ v = \frac{2GM}{R} \]

**Teacher notes:**

(kinetic energy = \( \frac{1}{2}mv^2 \) and gravitational potential energy at position R (based on potential energy at infinity equaling zero) = \( \frac{GMm}{R} \))

See appendix A for the calculus derivation if interested.

Students should practice with the formula derived in the teacher notes to calculate the escape speed for the earth and other planets.

See http://hyperphysics.phy-astr.gsu.edu/hbase/vesc.html

or give students the following data:

<table>
<thead>
<tr>
<th></th>
<th>Mass(^*)</th>
<th>Radius(^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.0558</td>
<td>0.383</td>
</tr>
<tr>
<td>Venus</td>
<td>0.815</td>
<td>0.95</td>
</tr>
<tr>
<td>Earth</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Mars</td>
<td>0.107</td>
<td>0.532</td>
</tr>
<tr>
<td>Jupiter</td>
<td>318</td>
<td>11.2</td>
</tr>
<tr>
<td>Saturn</td>
<td>95.1</td>
<td>9.41</td>
</tr>
<tr>
<td>Uranus</td>
<td>14.5</td>
<td>4.06</td>
</tr>
<tr>
<td>Neptune</td>
<td>17.2</td>
<td>3.88</td>
</tr>
<tr>
<td>Pluto</td>
<td>0.01</td>
<td>0.2</td>
</tr>
<tr>
<td>Moon</td>
<td>0.0123</td>
<td>0.273</td>
</tr>
</tbody>
</table>

\(^*\)relative to Earth

Earth mass = 5.976 \times 10^{24} \text{ kg}

Earth equatorial radius = 6378 \text{ km}

\[ G = 6.67 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2 \]
Also calculate the escape speed for:
1. Our Sun (M = 1.99 x 10^{30} kg, Radius 6.96 x 10^8 m)

2. A white dwarf with one solar mass but the radius of the earth.

_answers to escape velocity calculations above, to three significant digits:

<table>
<thead>
<tr>
<th>Planet</th>
<th>Escape Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>4270</td>
</tr>
<tr>
<td>Venus</td>
<td>10400</td>
</tr>
<tr>
<td>Earth</td>
<td>11200</td>
</tr>
<tr>
<td>Mars</td>
<td>5010</td>
</tr>
<tr>
<td>Jupiter</td>
<td>59 600</td>
</tr>
<tr>
<td>Saturn</td>
<td>35 500</td>
</tr>
<tr>
<td>Uranus</td>
<td>21 000</td>
</tr>
<tr>
<td>Neptune</td>
<td>32 500</td>
</tr>
<tr>
<td>Pluto</td>
<td>2500</td>
</tr>
<tr>
<td>Moon</td>
<td>2370</td>
</tr>
<tr>
<td>Sun</td>
<td>618 000</td>
</tr>
<tr>
<td>White dwarf</td>
<td>6 450 000</td>
</tr>
</tbody>
</table>

Ask students if they notice any patterns for these escape velocities.

Prompt them to compare similar radii with different masses such as Uranus and Neptune.

*Neptune’s escape speed is about 1.5 times that of Uranus because of the additional mass.*

Prompt them to compare similar masses with different radii such as the sun and a white dwarf.

*The white dwarf has almost 10 times the escape speed of the sun because of the much smaller radius. The factor of 1/100 the radius yields 10X the escape speed because the radius is in the denominator and because of the square root.*

Now ask students to calculate the escape speed from the gravity of the 3 galaxies in NGC 2300.
Mass = 6 X 10^{11} solar masses, R = 0.65 million light years

*The escape speed is 1.61 X 10^5 m/s (See Appendix C for calculation)*

Tell the students that the data from the images were taken with the Position Sensitive Proportional Counter instruments on ROSAT during April 25-27, 1992 give speeds that exceed the escape velocities calculated using the visible mass of the cluster of galaxies. The mass of the visible matter of the cluster (i.e. the galaxies & gas) isn’t enough to hold the gas.

Students will calculate the speeds of the hydrogen gas based on the ROSAT findings.
The x ray spectrum measured by ROSAT is best fit by a thermal model showing a temperature of approximately 10^7 K, which is an energy of 1 keV
Source: [http://www.astronomy.pomona.edu/astro121/mulchahey_araa.pdf](http://www.astronomy.pomona.edu/astro121/mulchahey_araa.pdf)
Explain that one electron volt in an energy unit is equivalent to the charge of one electron being raised through one volt of potential difference or $1.6 \times 10^{-19}$ Joules.

Setting this energy equal to kinetic energy and solving for the speed, the student can calculate the speed of the gas in the cloud in NGC 2300. Since the gas is hydrogen, use the mass of a proton or $1.67 \times 10^{-27}$ kg.

$$v = 4.377 \times 10^5 \text{ m/s}$$

See Appendix C for the calculation.

This is more than the calculation of the escape speed based on the galaxies alone. Something else must be contributing to the gravity, some mass that we cannot see with telescopes across many electromagnetic wavelengths.

This is evidence for a Dark Matter.

**III. Explanation**

Students will research the results further in order to understand the ideas in the engagement. Suggested sources (besides those listed in the teacher notes) include:

http://www.nasa.gov/home/hqnews/1993/93-001.txt
http://www.noao.edu/outreach/aop/observers/n2276.html
http://xrtpub.harvard.edu/xray_astro/dark_matter.html
http://hubblesite.org/newscenter/archive/releases/1993/05/text/
http://www.solarviews.com/cap/ds/rosat5.htm
http://home.eclipse.net/~cmmiller/DM/
http://www.astronomy.ohio-state.edu/~pogge/Ast162/Intro/gravity.html
http://www.physics.ucla.edu/~cwp/articles/rubindm/rubindm.html

Students will collect information in order to explain the findings documented in the news release and why the data indicates that dark matter is present.

Dark matter should be defined and ideas about it refined before the writing is begun.

Teachers should give intermediate assessments of student understanding of the topic by informal Socratic dialogs about their work.
Students may calculate the actual mass needed to produce the observed temperature based speeds by equating \( v^2 \) in the kinetic energy equation to \( v^2 \) in the escape velocity formula. The result is about 5 times the mass of the galaxies present, although these are fairly crude calculations and are less than the NASA findings.

\[
KE = \frac{1}{2} m_p v^2 \quad \text{so} \quad v^2 = \frac{2KE}{m_p}
\]

\[
v = \frac{2GM}{R} \quad \text{so} \quad v^2 = \frac{2GM}{R}
\]

Therefore:

\[
M = \frac{R \times KE}{m_p \times G} = 5.52 \times 10^{42} \text{ kg}
\]

**IV. Extension**

Students may draw a representation of the galactic cluster with galaxies, dust, gas and dark matter.

**V. Evaluation**

Students will write an article for their school newspaper that will explain the findings of ROSAT in April 1992 and the implications for the existence of Dark Matter. In addition they will tell about their experiences at the NASA conference in Washington DC.