Information Exchange on Newtonian gravity and Einstein's Theory of General Relativity

It is 1918, Einstein published his General Theory of Relativity just a couple years ago, and has yet to test it in a regime that will set it apart from Newtonian Gravity.

We want to exchange information on these two theories, especially in regards to how they account for the orbits of the planets in our Solar System. To that end, we will attempt to answer the following questions for each of the theories:

1. How was the theory developed?
2. Give a description of the theory.
3. How does it explain the motions of the planets?
4. How does it solve the problem of Mercury's orbit?
5. What prediction does it make for the bending of light and the upcoming eclipse in 1919?

1: Newton develops his theory

Newton wasn't hit on the head by an apple, but he may have watched one fall from an apple tree near his window. When he saw the apple fall, the light bulb that went off in his head was not that gravity existed – he and his contemporaries already knew that. Instead, Newton realized that gravity might extend so far above the Earth that it could be the force holding the moon in its orbit.

Newton published his law of universal gravitation on July 1687 in the Principia. With the Principia, Newton became internationally recognized.
2: *How Newton's gravity works*

Newton showed that if the force of gravity decreased as the inverse square of the distance, that he could calculate the Moon's orbital period. He postulated that this same force was responsible for other orbital motions, and so it was called "universal gravitation".

Newton's equation describing the gravitational attraction of two objects of mass $m_1$ and $m_2$ separated by a distance, $d$, is given by:

$$F_G = G \frac{m_1 m_2}{d^2}$$

where $F_G$ is the force of gravity and $G$ is the gravitational constant.

Newton's laws of motion have stood up to intense scrutiny and testing for over 200 years.

3: *Newton's gravity predicts the planet, Neptune*

Since ancient times, people have known about the planets Mercury, Venus, Earth, Mars, Jupiter and Saturn. In 1781, William Herschel, a comet hunter, discovered Uranus in 1781. In 1821, Alexis Bourvard published detailed tables of Uranus' orbit. However, observations showed deviations from these tables, leading Bouvard to hypothesize a perturbing body.

Independently, John Couch Adams (1845) and Urbain Le Verrier (1846) calculated an orbit for an 8th planet that would account for Uranus' motions.

The night in 1846 that Johann Gottfried Galle began observations to search for this new planet he found it! The planet was within 1° of where Le Verrier had predicted (and 10° of where Adams predicted). Observations over the next few nights confirmed that this new object was a planet, which was later named Neptune.
4a: Mercury's orbit

All planets orbit the Sun in an oval, as described by Kepler's Laws, so there is a point in that orbit that is closest to the Sun, called the perihelion.

We observe that as Mercury orbits the Sun, the perihelion advances by a small amount. Newton's theory does not fully explain this precession of Mercury's perihelion. There is a 43 arcsecond per century discrepancy between the Newtonian prediction (resulting from the gravitational tugs of the other planets) and the observed precession.

4b: How Newton's gravity explains Mercury's orbit

Newton's theory of universal gravitation has had wonderful success in explaining the motions of the planets in our Solar System, even leading to the discovery of Neptune. However, given the known planets in our Solar System, Newton's theory does not fully explain the precession of Mercury's orbit.

Some astronomers theorize that another planet, inside the orbit of Mercury, could account for the odd behavior of Mercury. This position is bolstered by the success in finding Neptune using the perturbations of Uranus' orbit to calculate an orbit of the undiscovered planet. The proposed planet between the Sun and Mercury has been named Vulcan, but has yet to be observed.
5a: Newtonian gravity bends light

Newton's theory of gravitation predicts that light will bend when traveling near a massive object – the larger the mass of the object, the larger the effect of bending.

In 1801, Johann von Soldner performed 25 pages of calculations to find that for Newtonian gravity the deflection angle of light passing near an object is:

$$ a \approx \frac{2m}{r} $$

where $m = \frac{GM}{c^2}$, $M$ is the mass of the sun, $r$ is the closest approach distance of the photon to the sun.

This solution is an approximation, since the full solution is an infinite sum of algebraic terms. The equation above gives the first (and largest) term of that series, which is a good approximation. The other terms of the series give small corrections to this solution, but those corrections would be too small to measure at the current time.

5b: Newtonian gravity's eclipse predictions

On May 29, 1919, there will be a total eclipse of the Sun. This eclipse is ideal for measuring the bending of starlight by the Sun. Totality will last for over five minutes – one of the longest eclipses in recent history. Also, the Sun will be directly in front of the Hyades, a bright cluster of stars in the constellation Taurus.

Newtonian gravity predicts that the angle of deflection will be 0.87 arcseconds for the stars near the Sun during this eclipse.

![Diagram showing the path of starlight during an eclipse](image)
1a: Einstein develops his theory

From his library in his Berlin apartment, Einstein saw a man fall from a nearby rooftop, fortunately onto a pile of soft rubbish. When the man described the event to Einstein, he said that he did not feel the sensation of gravity as he fell, which he would have expected to pull him violently downward.

While Einstein was extending his special theory of relativity, which applies to systems undergoing uniform motion, to systems with non-uniform motion (what he calls difform motion), he remembered the falling man, and realized that gravitation might be described as a "difform motion" (non-uniform motion).

He published his general theory of relativity in 1915.

1b: Einstein on how his gravity differs from Newton's

From an interview printed in the New York Times, Dec 3, 1919, page 19:

[Quoting Einstein]
"Please imagine the earth removed, and in its place suspended a box as big as a room or a whole house, and inside a man naturally floating in the centre, there being no force whatever pulling him. Imagine, further, that this box begin, by a rope or other contrivance, suddenly jerked to one side, which is scientifically termed 'difform motion,' as opposed to 'uniform motion.' This person would then naturally reach bottom on the opposite side. The result would consequently be the same as if he obeyed Newton's law of gravitation, while, in fact, there is no gravitation exerted whatever, which proves that difform motion will in every case produce the same effects as gravitation."

The concept Einstein describes above is known as the Equivalence Principle.
2: How Einstein's gravity works

Einstein's theory proposes that space and time are one entity – spacetime. Any object with mass will curve spacetime, with the mass of the object determining the amount of curvature produced. Free-falling objects follow "geodesics", or the shortest path through the curved spacetime. We observe objects attracted to massive objects as they follow these paths. We call this gravity.

Einstein's gravity becomes evident only in extreme conditions, such as near very massive objects, which cannot be replicated in today's laboratories. Because of this, the theory has yet to be put to any rigorous testing.

3: Einstein gravity explains the planetary motions

The Sun is the largest object in our Solar System, so, according to Einstein's General Theory of Relativity, the Sun will "warp" spacetime in the Solar System by the largest amount. The planets simply follow the shortest path through spacetime around the Sun, thus explaining their orbits.
4a: Mercury's orbit
All planets orbit the Sun in an oval, as described by Kepler's Laws, so there is a point in that orbit that is closest to the Sun, called the perihelion.

We observe that as Mercury orbits the Sun, the perihelion advances by a small amount. Newton's theory does not fully explain this precession of Mercury's perihelion. There is a 43 arcsecond per century discrepancy between the Newtonian prediction (resulting from the gravitational tugs of the other planets) and the observed precession.

4b: How Einstein's gravity explains Mercury's orbit
Albert Einstein’s General Theory of Relativity provides a full explanation for the observed precession of Mercury's perihelion without the need for an extra planet.

As Mercury moves toward its perihelion (i.e. closer to the Sun), it moves deeper into the Sun's gravity well. Its motion into this region of greater curvature of space-time causes the perihelion to advance. Einstein's Theory of General Relativity predicts exactly the amount of perihelion advance seen in Mercury. Einstein views this result as the most critical test of his theory to date.
5a: Einstein gravity bends light

Einstein's theory of gravitation predicts that light will bend when traveling near a massive object - the larger the mass of the object, the larger the effect of bending.

In 1915, using his own equations of general relativity, Einstein calculated that angle to be:

\[ a \approx 4m/r \]

where \( m = GM/c^2 \), \( M \) is the mass of the sun, \( r \) is the closest approach distance of the photon to the sun.

This solution is an approximation, since the full solution is an infinite sum of algebraic terms. The equation above gives the first (and largest) term of that series, which is a good approximation, given the accuracy of the measurements possible at this time.

5b: Einstein gravity's eclipse predictions

On May 29, 1919, there will be a total eclipse of the Sun. This eclipse is ideal for measuring the bending of starlight by the Sun. Totality will last for over five minutes – one of the longest eclipses in recent history. Also, the Sun will be directly in front of the Hyades, a bright cluster of stars in the constellation Taurus.

Einstein's General Relativity predicts that the angle of deflection will be 1.75 arcseconds for the stars near the Sun during this eclipse.